Category Theory for Language 2

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The philosopher Ludwig Wittgenstein (from 1930 onward according to "A Wittgenstein Dictionary") used a game analogy to explain language, famously referring to language use as language-games. Apparently, he liked chess. Let's take a look at the game of chess for a moment. There must be millions of chess pieces in the world, all different shapes and sizes, but each piece is of a certain type, a type identified not by its shape or size but by its name and **moves it may make on a chess board**. There are six different types of chess piece - pawn, rook, knight, bishop, queen, and king. With 32 of these six types of chess piece there are an *estimated* 10^{120} possible games of chess (the "Shannon Number").

In my previous small paper "Category Theory for Language" I talked about a fundamental building block for a vocabulary, a category or bunch of words that have the same "meaning in use." But we need understand relationships between these building blocks and perform functions on them to make sense of what people say. So, we treat these categories as objects in another category, the category of all the words an individual (or conversational robot) can use, the individual's (or robot's) vocabulary. Do all the words a person (or robot) can use form a category? Yes, after we apply another abstraction. Here are the three axioms of a category with respect to that vocabulary:

abstraction - Using abstraction, we remove unnecessary details, and the only detail to be removed is *what* each object (bunch of words) means in use. With abstraction we remove what they mean, and all we are left with is that they have a "meaning in use."

identity(isomorphism) - All the words in the vocabulary are interchangeable in that they all have a "meaning in use."

composition - For example, if object 1 has a "meaning in use" as does object 2, and object 2 has a "meaning in use" as does object 3, does object 1 have a "meaning in use" as does object 3? Yes. Composition is satisfied. (Associativity is satisfied too.)

Now that we have identified our vocabulary as a category, we apply another step inspired by the game of chess.

Say there are six different types of "meaning in use" as there are six different types of chess pieces. These types are defined by the abilities of our category's objects in the real world, just as chess pieces are defined by the moves they may make on a chess board. The types' abilities can be defined as "tells what happened" or "asks question" and so on. To create six types, we apply six morphisms (monomorphisms) to our vocabulary category.

So that's it, a vocabulary described with category theory. What can we do with this vocabulary? Can a person and a conversational robot have 10^{120} different conversations? I believe they can. Functions can use the vocabulary to save what a person says in a readable format, generate responses, read what was said previously in a conversation and so on.